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Late Medieval Discussions of the Continuum and the Point of the Middle English Patience

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> "We are not *talking* about gorse-bushes," said Owl a little crossly.
> "I am," said Pooh.
>
> A. A. Milne

The poet of *Patience* probably did not realize the strain he placed on glossators and lexicographers by his use of the word "point":

> Pacience is a poynt, þaȝ hit displese ofte.[1] (1)

Anderson, the most recent editor, suggests in a note that "poynt" should be taken "as a noun meaning 'good quality, virtue', an unusual meaning, but cf. *GGK*. 654: 'pite þat passeȝ alle poyntes' ".[2] In *Sir Gawain and the Green Knight* line 654 occurs in the passage describing Gawain's shield and its virtues,[3] where the points and sides of the

[1] Quotations from *Patience* are taken from J. J. Anderson, ed., *Patience*, Manchester 1969, and are noted parenthetically in the text.

[2] Anderson, p. 50. Richard Morris, ed., *Early English Alliterative Poems*, EETS 1, London 1864, does not comment on the word either in his notes or in his glossary. The second edition (London 1869) glosses "point" as "particle", with a reference to *Pearl* 891, but again has no note on line 1 of *Patience*. Hartley Bateson, ed., *Patience* Manchester, 1912; 2nd ed., *ibid.*, 1918 glosses it simply as "matter, point" in both editions. In the first he attributes the emendation of line 1 (Patience is a [nobel] poynt) simply to a suggestion of G. C. Macaulay's without saying where Macaulay makes the suggestion. In the second edition he gives an extensive note in which he glosses "point" in lines 1 and 531 as "matter, item", justifies his emendation to "[nobel] poynt" on lexical grounds, rejects Gollancz's gloss, "an essential thing", on the grounds that it abuses the idiomatic expression "*the* point", and reluctantly rejects the suggestion of O. F. Emerson, *Two Notes on* Patience, in: MLN, 29 (1914), 85, to read "apoynt" and to gloss "commanded or prescribed". Israel Gollancz, ed., *Patience* (London: Oxford 1913), glosses "point" as "essential thing, point, matter" and in his notes observes with acerbity that there is no reason for supplying "nobel" before "poynt" in line 1. In the notes to his second edition (London: Oxford 1924) he again rejects Bateson's emendation to "nobel poynt" and in his glossary repeats his earlier definition.

[3] J. R. R. Tolkien and E. V. Gordon, eds., *Sir Gawain and the Green Knight*, 2nd ed. rev. by Norman Davis, Oxford 1967, lines 619-69.

pentangle are delineated and explained. It is true that the poet says of pity that it "passeȝ alle poynteȝ", but in this context "poynte" may well mean "points of the pentangle" with their attendant virtues rather than simply "virtues" or "good qualities" alone.

In the extensive article on "point" in the *N.E.D.*, no definition corresponds with Anderson's. One, A 24, comes reasonably close: "A (specified) degree of condition; condition, plight, state, case (good, evil, better, etc.) (Cf. F. *en bon point.*)" with citations including Robert of Gloucester (1297), *Alexander and Dindymus* (1340-70), and Chaucer's *General Prologue.* The *Patience*-poet himself seems to use the word in this sense at *GGK* 2049 (Hym lyst prik for poynt, þat proude hors þenne) and perhaps at *Patience* 35 (Bot syn I am put to a poynt þat pouerte hatte), though in neither case does he include the qualifying adjective. But the meaning "condition" or "plight" fits lines 1 and 531 of *Patience* only with some awkwardness or with the stretch of meaning suggested by Anderson.[4]

The concordance to the *Patience*-poet's works gives some twenty instances of "point" and "points".[5] It seems to me that in almost every case the sense of the word as used in its context presents few lexical problems, and each sense corresponds to one of those given in the *N.E.D.* We may infer that for Anderson's interpretation to be correct, the poet would need to use the word in a way apparently foreign to his normal usage.

I should like to propose an explanation that is at once simpler and more complicated: simpler in that it relies on one of the standard meanings of "point" but more complicated in that it will draw us into scholastic discussion of rather abstruse matters. If we take "point" to mean a dot, a geometrical figure of no dimensions (i.e., a Punkt rather than a Spitze), then the *N.E.D.* clearly supports the meaning without any stretching. But if patience is a dot or a geometrical figure of no dimensions, then it is intelligible only by reference to discussions of the continuum which occupied many philosophers in the universities during the thirteenth and fourteenth centuries.

It is not improbable to suppose that the poet had some access to

[4] The *Middle English Dictionary*, ed. Hans Kurath, Sherman M. Kuhn, et al. (Ann Arbor 1952-), was complete only through *metal* as of November 1976, when this paper was written.

[5] Barnet Kottler and Alan M. Markman, *A Concordance to Five Middle English Poems*, Pittsburgh 1966, p. 442. I have not counted variants. The frequency list on p. 717 gives only fourteen instances of "poynt", though the list on p. 442 clearly provides fifteen.

these discussions, probably at a university. His grasp of theology in *Pearl*, though it may have been learned elsewhere, allows us to infer without undue fantasizing some university training. Moreover, in *Patience* itself there may well be an allusion to a period spent at university. When Jonah prays for deliverance from inside the whale, he says:

> I haf meled with þy maystres mony longe day,
> Bot now I wot wyterly þat þose vnwyse ledes,
> Þat affyen hym in vanyte and in vayne þynges,
> For þink þat mountes to noȝt her mercy forsaken. (329-32)

These masters, the usual title for a university teacher in the Middle Ages, are no more than a simple "qui" in the Vulgate: "Qui custodiunt vanitates frustra, misericordiam suam derelinquunt" (Jonas, 2:9). Perhaps the vanitates frustra, the vanyte and vayne þynges, reminded the poet of some of the super-subtle distinctions and determinations he had heard at university.[6] He was certainly not alone in feeling repulsed by the subtleties of university thought: Richard Rolle also left Oxford without taking a degree,[7] and in his writings he alludes frequently to the vanity of over-refined thinking.[8] Even Richard Fitz-Ralph, who did earn university degrees and who spent some time as a university master, ultimately rejected the subtlety of scholastic thought in favor of a more direct relation with God.[9] The *Patience*-poet may well have used a university lecture hall as the starting place for *Patience*. Not that the poem participates in the debate after the mode of the debaters, for it does not, but rather it turns university concepts into images and, despite its use of philosophical terms, does not allow the philosopher any proper entrée.

Points were a matter of concern to fourteenth-century philosophers only to the extent that they were said to be the units of which a line was made. When considered from this point of view, the concern over the nature of points stretches back into antiquity to Democritus, Zeno,

[6] See, for example, William J. Courtenay, *John of Mirecourt and Gregory of Rimini on Whether God Can Undo the Past*, in: Recherches de théologie ancienne et médiévale, 39 (1972), 245.

[7] Hope Emily Allen, *Writings Ascribed to Richard Rolle Hermit of Hampole and Materials for his Biography*, New York 1927, pp. 55-56.

[8] Several passages against such vanities are collected by F. M. M. Comper, *The Life of Richard Rolle*, New York 1928, pp. 46-49.

[9] A brief account is given by J. A. Robson, *Wyclif and the Oxford Schools*, Cambridge Studies in Medieval Life and Thought, n.s. 8, Cambridge 1961, pp. 90-92.

and Aristotle and beyond the Middle Ages at least to Cavalieri in the seventeenth century.[10] For our present purposes, it is perhaps sufficient to restrict the discussion to a general view of the problem and its development, along with a sample of some fourteenth-century philosophical pronouncements on it.

For Aristotle, who was the source of medieval knowledge of the problem, the continuum is the fundamental structure of the physical world, whether it is a matter of space (point, line, surface, solid), movement (change of quality, quantity, or place), or time.[11] Moreover, every continuum is for Aristotle, as I suspect it would be for modern man, infinitely divisible. In the version of the *Physics* that became available to European philosophers in the thirteenth century, Aristotle says, "It is impossible for any continuum to be made up of indivisible parts".[12] His objection rests on two difficulties that he perceives in the joining together of indivisibles: first, if an indivisible is joined to another, the joining must be on a side in order for them to become a continuum; but if an indivisible has a side, then it has a part separable from another part and thus is not indivisible. On the other hand, if indivisibles are not joined on the sides, then no accumulation of them, however great, can amount to more than one, since all those that are added will simply be added on top of the first one.[13]

So much was passed to medieval thinkers when the *Physics* became available, and even though many of the thirteenth-century commentators chose not to follow Aristotle's view of the matter, no controversy seems to have arisen as a result.[14] Controversy comes in the early

[10] For a history of this question, see Wolfgang Breidert, *Das aristotelische Kontinuum in der Scholastik*, Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters, N.F. 1, Münster i. W. 1970; John E. Murdoch, *"Rationes mathematice": un aspect du rapport des mathématiques et de la philosophie au moyen âge*, Conférence donnée au Palais de la Découverte le 4 novembre 1961, Paris 1962; V. P. Zoubov, *Jean Buridan et les concepts du point au quatorzième siècle*, in: Medieval and Renaissance Studies, 5 (1961), 43-95; and V. P. Zoubov, *Walter Catton, Gérard d'Odon et Nicolas Bonet*, in: Physis, 1 (1959), 261-78.

[11] See Breidert, p. 10.

[12] Impossible est ut aliquod continuum sit compositum ex indivisibilibus, *6 Physicorum*, t. c. 1 (231[a] 24), in *Averrois commentaria et introductiones in omnes libros Aristotelis cum eorum versione latina* (Venice: Iuntas, 1562-74), vol. 4, fol. 246[v] K, quoted in John E. Murdoch and Edward A. Synan, *Two Questions on the Continuum: Walter Chatton (?), O.F.M. and Adam Woodham, O.F.M.*, in: Franciscan Studies, 26 (1966), 212-88; the quotation is on p. 234.

[13] See W. D. Ross, ed., *Aristotle's Physics*, Oxford 1936, 6. 231[a] 21ff. and p. 402.

[14] Murdoch, *"Rationes mathematice"*, believes that the atomistic or indivisibilist view gained adherents only in the fourteenth century. But see Breidert, chapters III and IV, who shows the atomism of Albert the Great, Giles of Rome, Thomas

fourteenth century, and although from six centuries remove it is difficult to say why or even how it got started, it does seem as if four or five names are involved in its beginnings: Henry of Harclay and Walter Catton in England, Gerard of Odo, Nicholas Bonettus, and John the Canon on the continent.[15] Breidert speculates that its origins lie in the Atomists' dismay over the prospect of actual infinity: if a line really were infinitely divisible, then not even God would know exactly how many parts it had.[16] This problem, however, seems to me perennial, since it can be found as early as Grosseteste and as late as Wyclif. The fact that it can also be found in the work of Walter Catton and John the Canon may indicate an intensification of concern in the early fourteenth century, but the evidence I have located does not offer overwhelming support for such a view.[17]

Aquinas, and Robert Grosseteste. See also Richard of Middleton's commentary on the *Sentences*, where he uses the relation of points to a line in order to argue the nature of the union of the divine and the human in Christ: Quod tamen ibi supponitur, quod punctus per divinam potentiam non posset separari a linea non credo verum esse. *Clarissimi theologi magistri Ricardi de Mediavilla . . . super quattuor libros sententiarum* (Brixiae 1591), Book III, dist. 6, art. 3, q. 3, ad 2 (p. 66).

[15] This theory of origins is Murdoch and Synan's, pp. 212-15, who go on to show some of the lines linking various antagonists. John the Canon is difficult to date and may properly belong to a later period in the first half of the fourteenth century.

[16] p. 36. Additional speculations are in Murdoch and Synan, p. 215, fn. 11.

[17] Grosseteste says, Et iterum audacter dico quod omnis numerus infinitus ipsi deo cuius sapiencie non est numerus infinitus plus quam binarius, qui est mihi infinitus est illi finitus: . . . sicut enim que vere in se finita sunt, nobis sunt infinita, sic que vere in se infinita sunt, illi sunt finita. Robert Grosseteste, *Commentarius in octo libros physicorum Aristotelis*, ed. Richard C. Dales, Boulder 1963, p. 92. Catton: tertia conclusio est quod licet nulla creatura, per leges adhuc concessas, possit complere divisionem continui, nec dividere minutim quin semper pars divisa habet partes, nec eciam videre unam rem indivisibilem, tamen, deus potest, et posset conferre creature potentiam, quare non ita bene sicud ignis materialis agit in animam, et posset facere quod occulus videret spiritum? Murdoch and Synan, p. 247, paragraph 58. Wyclif: Sed ad illud respondet Lincolniensis, dicens quod . . . apud Deum est notum ex quot non quantis mundus componitur, et quecunque pars eius corporea; sicut notum est sibi quota pars quantitativa anguli recti est angulus contingencie . . . Johannis Wyclif, *Tractatus de Logica*, ed. M. H. Dziewicki, 3 vols., London 1893-99, cap. ix, q. 2; vol. III, p. 35. John the Canon also reports that this view exists: Circa sextum huius quero hanc questionem. utrum quodlibet continuum componitur ex divisibilibus et resolvatur in semper divisibilia. Et videtur quod non. Primo quia sequeretur quod in continua essent alique partes a deo incognite. consequens est falsum ergo antecedens. falsitas consequentis patet quia deus nihil ignorat. aliter esset imperfectus. Subsequently he goes on to say that he is unable to resolve the problem: Ex hiis igitur que dicta sunt in secundo huius questione de contingentia rerum tractatum est utrum omnes partes continui sint a deo

But whatever the nature of and reasons for its origins, attitudes and defenses of attitudes during the fourteenth century fall roughly into three categories: the loosely philosophical, the semantic, and the mathematical. In the loosely philosophical category the Atomists, who defend the indivisibility of the point, and the Infinitists, who defend its infinite divisibility, seem capable of mustering convincing arguments in support of their respective positions, but in the mathematical and semantic categories the Infinitists have a clear and decisive upper hand. The reason for this is perhaps that the distinction between the abstract and physical natures of the question were not always recognized or observed,[18] but semantics and mathematics provided comparatively unambiguous means for clearing away ambiguities. The basic question around which the discussion revolved was whether or not the parts of a continuum—the points of a line, the lines of a surface, the surfaces of a solid—were actually real. Everyone was apparently willing to grant that such things formed a part of everyday language and were useful to geometers, but whether they actually existed or were merely convenient fictions was a thornier problem.

The semantic view of the Infinitists was that they did not really exist but that they were harmless enough and even useful when properly understood. William of Ockham on many occasions stresses the need to see what an author really means rather than simply to show by ingenuity that his words are foolish. In discussing this question too he takes the same point of view: what is really meant by a point is simply that the line extends no further, and thus the term "point" signifies no more than a deprivation of further material of the same sort.[19] John Buridan, writing only a few years later, follows Ockham

cognite. Ex hiis eciam que dicta sunt in tertio huius questione de infinito. et ex hiis que nunc dicta sunt recollige hanc materiam: et in hac materia omnes locuntur dubie: et nunquam ab alio doctore potui istam materiam bene intelligere. *Questiones Johannis Canonici super octo libros physicorum aristotelis* (Sancti Albani 1481), Book VI, q. 1, fol. t 4^{v} and u 2^{r}.

[18] John the Canon, for example, recognizes the distinction: Prima distinctio est ista. quod continuum potest accipi dupliciter uno modo mathematice. alio modo naturaliter. primo modo sumitur sub ratione praecisa qua est quoddam quantum abstractum per intellectum ab omni quantitate sensibili et ab omni motu et materia naturali. secundo modo sumitur sub racione[m] que est coniunctum istis et habet naturam que est principium motus naturalis. *Questiones Johannis Canonici*, Book VI, q. 1, verso of the fourth folio after fol. t 4. See also Zoubov's comments in *Walter Catton et al.*, p. 276.

[19] Unde sicut dicunt aliqui quod hoc nomen privatio equivalet in significando huic toti: subiectum privatum et propter hoc sicut concedunt istam subiectum privatum est materia ita concedunt illam privatio est materia. Ita si hoc nomen

in this semantic distinction, drawing a comparison with the word "blindness". Just as blindness does not denote a thing in itself but rather a lack of vision, so too a point means simply a lack of extension or a lack of divisibility.[20] And essentially the same argument can be found still intact late in the fourteenth century in the work of Albert of Saxony, one of Buridan's pupils who survived the plague of 1348-49.

punctus in significando equivalet isti toti linea non ulterius protensa vel alicui consimili cuius una pars sit linea et alie partes sint aliqua adiectiva vel verbum coniunctum cum linea mediante hoc pro nomine que vel altero consimili sicut hec conceditur: linea est divisibilis ita illa conceditur de virtute significationis punctus est linea. punctus est divisibilis. Quicquid enim verificatur de hoc nomine linea sumpto significative verificatur de hoc nomine punctus sumpto significative et hoc de virtute significationis loquendo . . . . Nec est inconveniens concedere quod philosophi et sancti sic figurative loquebantur cum multa talia in sacra scriptura reperiantur. non tamen accipiendo dicta sacre scripture in sensu spirituali: sed etiam in sensu literali. immo etiam frequenter vulgaris et communis modus loquendi talibus figurativis locutionibus utitur sicut per dicta grammaticorum de talibus tractantium potest probari. Unde non tantum dictio potest transferri a propria significatione ad impropriam sed etiam aliquando dantur vocabula que nullam propriam significationem habent qualem primo instituta habent. Et sicut translatio secundum grammaticos fit triplici de causa: quia aliquando causa metri sicut in poetis, aliquando causa ornatus sicut in rhetorica locutione. aliquando causa necessitatis sive brevitatis sive utilitatis sicut in philosophia et omnibus istis modis fit translatio in theologia . . . . quamvis ex predictis causis occasionaliter multi errores proveniunt in simplicibus que omnia dicta philosophorum et sanctorum secundum proprietatem sermonis volunt accipere, cum tamen sint figurative sumenda . . . . patet quod "ille punctus est terminus linee" est falsa de virtute sermonis, est tamen vera secundum sensum quod pretendit sicut dictum est. *Tractatus venerabilis inceptoris Gulhelmi Ockam de sacramento altaris* (Paris 1513), q. 1, conclusio. Compare the *Expositio Aurea*, II: punctus est nomen vel intentio importans non ulteriorem pertransitionem secundum longitudinem, quoted by E. A. Moody, *The Logic of William of Ockham*, London 1935, p. 147.

[20] Ad evidentiam solutionis istarum omnium questionum sciendum quod hoc nomen *punctum* est nomen privativum, sicut *cecitas*. Nam sicut *cecitas* significat carentiam visus in oculo, ita *punctum* significat carentiam divisibilitatis seu extensionis in magnitudine . . . . Dicere quod linea terminata est ad punctum vel quod punctum terminans est in linea, nihil aliud significat quam dicere, quod linea ultra certam quantitatem non extenditur, sed sicut dicere quod linea ultra certam quantitatem non extenditur, sed sicut dicere quod oculus est cecus vel quod cecitas est in oculo nihil aliud significat quam quod in oculo non est visus. *Questio de puncto*, III, 1, in Zoubov, *Jean Buridan*, p. 86. In his commentary on the *Physics*, he makes a similar point, that the convenience of the term "point" does not imply its real existence: Respondetur quod hoc non dicitur quia sit ita vel quia sit verum de virtute sermonis, sed uno modo hoc dicitur secundum imaginationem mathematicorum, ac si esset punctum indivisibile non quia debeant credere quod ita sit sed quia in mensurando revertuntur eedem conditiones sicut si ita esset. *Acutissimi philosophi reverendi Magistri Johanis buridani subtillissime questiones super octo phisicorum libros*, Paris 1509, Book VI, q. 4, fol. 97^{r}-97^{v}.

Albert too, almost like Ockham's echo, argues that the word "point" is a "nomen privativum", indicating a lack of divisibility or the end of a magnitude.[21]

Since the Infinitists are perfectly willing to grant that the word "point" is a useful fiction, available to mathematicians, geometers, and indeed anyone else who wants to use it, the Atomists are at something of a disadvantage. What use is it to protest that a point is real, when your opponent is perfectly willing to concede it all the appearances of reality? Still Nicholas Bonettus proposes that a point really exists and is not simply the lack of a further line, but his analogical argument fails before the semantic arguments just cited.[22] In fact Nicholas, finding that the authority of Aristotle himself is opposed to his position, attributes Aristotle's own views to a certain perverse need to contradict almost everything that Plato ever said.[23]

Only Walter Catton, a contemporary of Ockham's at Oxford, was able to mount any sort of serious argument against the semantic criticism of the Atomist position. When read from a twentieth-century point of view, Walter's argument may seem a little absurd. Essentially it boils down to the proposition that if a line does not end in a finite and real point, then it cannot end at all. Since it is obvious that many lines do in fact end, then clearly, in Walter's view, each of them must end in a point. A point, therefore, is a real thing and not merely a

[21] Pro quo est notandum quod hoc nomen punctus est nomen privativum significans carentiam divisibilitatis non simpliciter: sed cum distributione facta per hoc nomen totum quod distribuit partes quantitativas. Et secundum hoc: hoc nomen punctus equivalet huic orationi magnitudo, et non tota magnitudo prout ly totum distributive tenetur. et ideo quandoque predicatum est verificabile de ista oratione magnitudo et non tota magnitudo, est verificabile hoc nomen punctus. Et ideo exquo magnitudo est terminus alicuius magnitudinis: et tamen nulla tota hoc est nulla secundum quamlibet eius partem. Ita dicimus punctus esse esse terminum magnitudinis. *Acutissime questiones super libros de Physica auscultatione ab Alberto de Saxonia*, Venice 1516, Book VI, q. 1, fol. 65^{r}, with corrections from the Venice, 1504, edition, fol. 64^{v}.

[22] Per punctum autem intelligo quandam entitatem positivam absolutam et indivisibilem, et non solum carentiam linee. Nam sicut linea est quoddam positivum indivisibile secundum latitudinem, et superficies quoddam positivum indivisibile secundum profunditatem. Sic et punctus est quoddam positivum simpliciter indivisibile. Nicholas Bonettus, *Predicamenta*, Book II, in *Nicholai Bonetti viri perspicacissimi quattuor volumina: Metaphysicam videlicet, naturalem philosophiam, predicamenta, necnon theologiam naturalem*, Venice 1505, fol. 78^{v}.

[23] Adhuc autem Aristoteles posuit quod continuum erat compositum ex infinitis partibus divisibilibus in infinitum, et hoc invehendo contra platonem et democritum qui ponebant quod continuum erat compositum ex finitis partibus indivisibilis. Et fertur quod Aristoteles hoc gratis posuit: quia quasi in omnibus voluit contradicere platoni. *Ibid.*, fol. 79^{v}.

convenient fiction.[24] Walter's view here depends upon taking only a part of the Infinitist argument and applying it where it was not designed to fit, but I have not found any Infinitist who attempts a refutation.

The conclusion seems inescapable that, as far as semantic distinctions are concerned, the Atomists do not win the day. Nor do they do very well with the mathematical arguments. These arguments, which originate with Euclid, were introduced to western Europe via al-Ghazzali's *Maqasid*, a digest of some of Avicenna's ideas.[25] Essentially, they are designed to reduce the Atomists' arguments to absurdity by showing that if the number of points in a line is really finite, then the diagonal of a square is equal in length to one of its sides; of two concentric circles, the smaller is equal in size to the larger; a line bisecting the sides of a triangle is equal in length to its base, and so on.[26] Since the tradition in this mathematical matter is remarkably constant throughout the fourteenth century, perhaps a few examples will suffice.

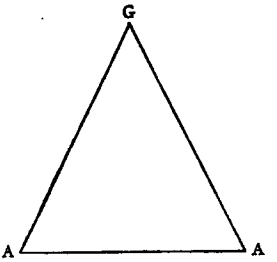
Marsilius of Inghen suggests that one assume for the sake of argument that a continuum is composed of indivisibles and construct a

[24] Here are two of the four arguments Catton gives: hic est contradiccio quod ultra aliquam multitudinem rerum non sit ultra accipere secundum aliquam divisionem, nec secundum locum illa re; sed sic sunt partes illius multitudinis finite et, tamen, quod partes illius multitudinis sunt intra se infinite, ita quod semper extra unam partem, vel preter unam, est sumere aliam in infinitum. probacio, quia, ut videtur, secundum dictum infert oppositum primi dicti, scilicet, quod, si sunt infinite intra quod sint infinite sic quod semper ultra secundum locum est aliquid accipere. probacio quid, si intra sint infinite, igitur, si omnes partes disponerentur secundum longum solum quantum possent, semper aliquid ultra sumere secundum longum, quia habent infinitam longitudinem. vel, si non, propositum. Murdoch and Synan, p. 242, paragraph 41. A second similar argument: illa res que finit partes istius rei, sic quod ultra non est protendere in longum istam rem, vel est tantum finita vel infinita. si finita tantum, propositum, quod indivisibilis, vel in indivisibilia resolubilis; si infinita, igitur non finit intrinseca quia illud quod habet infinitas partes non finit alias infinitas partes, quid quod in se non finitur, non finit alia, excepto deo infinito, qui finit omnia. Murdoch and Synan, p. 243, paragraph 45.

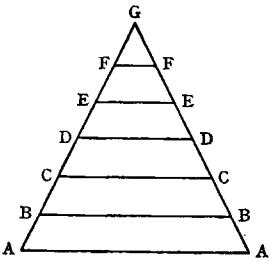
[25] See Murdoch, "*Rationes mathematice*", for a brief account of the introduction of these ideas to the west.

[26] Scholars agree that the fullest and mathematically most advanced use of this set of proofs is Thomas Bradwardine's *De continuo*, of which no edition is available. John E. Murdoch has had the work in hand for some time now and apparently made his draft available to Breidert, but I have not yet seen any notice of its being in print. His "*Rationes mathematice*" summarizes some of Bradwardine's proofs, and there is a very brief excerpt from the *De continuo* in Marshall Clagett, *The Science of Mechanics in the Middle Ages*, Madison 1959, pp. 230-34. Also some of Bradwardine's conclusions are quoted in V. P. Zoubov, *Walter Catton et al., passim.*

line, AA, made up of six points. Then on the line AA erect an equilateral triangle whose two sides join at point G, thus:



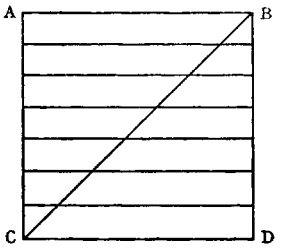
Then the two lines AG should be divided at their six points and lines drawn across connecting the pairs of points in the following manner:



Since both A and A are points and the line AA is made up of six points, there remain four points separating point A from point A. If four points separate A from A and BB is clearly shorter, then three points must separate B from B. And if that is so, then two points must separate C from C, and one point D from D. But EE and FF are shorter than DD and, therefore, must be less than one point. Therefore, a point is divisible.[27]

[27] Tunc ponitur ista conclusio: quod nullum continuum est compositum ex indivisibilibus. Probatur, quia si sic, sequeretur quod esset dare minus indivisibili, consequens est falsum et consequentia probatur. Sit AA, una linea sex punctorum, gratia exempli, super quam per corollarium primi Euclidis, constituatur triangulus duorum aequalium laterum, quorum quodlibet, gratia exempli, sit trium punctorum, et concurrant ista duo latera in puncto G, tunc in ambabus lineis A, G, signet punctum B, quod distet ab A per duo puncta, et protrahatur linea BB, deinde in eisdem lineis signetur punctum C, quod distet a B, per duo puncta, deinde D, quod distet a C per tantum, postea E, et sic consequenter usque ad G; tunc isto posito arguitur sic: puncta B minus distant ab invicem quam puncta A; sed puncta A distant per quattuor puncta, quia tota linea A ponitur sex punctorum; igitur puncta B, ad minus distant per tria puncta. Et iterum puncta C, minus distant quam puncta B; igitur ad minus distant per duo

Another mathematical proof that recurs is one that demonstrates that if one assumes that a point is indivisible, then the diagonal of a square is equal in length to the side. To demonstrate this, one constructs a square, ABCD, with a diagonal, CB. From several points on one side, let us say six, let a line be drawn to the other side parallel to the base of the square, thus:



For each point on the side of the square (either AC or BD), there is clearly a point on the diagonal. Thus there are as many points on the diagonal of the square as on its side, and thus the diagonal and the side are of the same length.[28]

Finally, let us assume a line of three points and two moving bodies, one twice as fast as the other, moving along the line. When the faster body has moved three points, the slower body will have to have moved a point and a half. Therefore, points are divisible.[29]

puncta. Item puncta D minus distant, quam puncta C, igitur ad minus distant per unum indivisibile; sed puncta E minus distant, quam puncta D, igitur minus quam per indivisibile; et habetur propositum conclusionis. *Questiones subtilissime Johannis Marcilii Inguen super octo libros phisycorum secundum nominalium viam*, Lugduni 1518; repr. Frankfurt 1964, Book VI, q. 1, fol. 64^{r}, with corrections from Johannis Duns Scoti, *Opera Omnia*, ed. T. Wadding, vol. III, Paris 1891, p. 251. Despite its place among Scotus' works, the commentary belongs to Marsilius.

[28] Dico quod nulla pars linee est indivisibilis, nec etiam pars quaecumque cuiuscumque continui est indivisibilis. Quod probatur sic: quia si sic, tunc costa quadrati esset equalis diametro, et sic esset diameter commensurabilis coste. Consequentia patet, quia a quolibet puncto diametri ad costam continget protrahere rectam lineam. Quod probatur, quia a quolibet puncto coste unius ad quodlibet punctum coste alterius contingit protrahere rectam lineam. Imo ita esset de facto posita ypotesi. Et quelibet talis linea protrahitur per aliquid punctum diametri; ergo a quolibet puncto diametri ad costam est aliqua linea recta. Si ergo sint sex puncta in diametro, erunt necessario sex puncta in utraque costa. William of Ockham, *Quodlibeta septem*, Argentine 1491, I, q. 9. Cp. Murdoch and Synan, p. 255, paragraphs 87, 88.

[29] Secundo sequeretur quod indivisibile esset divisibile. Consequens implicat. consequentia probatur: posito quod lineam factam ex tribus indivisibilibus per-

The Atomists did their best to refute these and similar mathematical proofs that were posed against their theories, but for the most part their attempts demonstrate very odd mathematics indeed and are not very convincing. Catton's refutation of the proof showing concentric circles to be of the same size [30] is ingenious enough. He suggests that the origin of the two concentric circles is two straight lines of equal length with points corresponding to each other. When these lines are curved into concentric circles, however, the inner one must lose a bit of itself in order to make a smaller circle. Therefore, its points will no longer be in line with those on the outer circle.[31] If they cannot be expected to be in direct linear relation with each other, then proofs that

transeat mobile velocius in aliquo tempore: et ponatur quod in eodem tempore super eandem lineam moveatur mobile in duplo velocius: igitur duplum spacium. pertransibit igitur punctum cum dimidio. Marsilius of Inghen, *op. cit.*, fol. 64^{r}, with corrections from Wadding's Scotus, p. 250.

[30] While it is not difficult to understand how this proof works, it is difficult to find a text that formulates it without ambiguity. Suppose two concentric circles, the outer of which consists of three points. The radius lines from each of these points are drawn to the center of the circle, cutting the inner circle at three points in line with those on the outer circle. It then appears that the inner circle is also of three points and, therefore, the same size as the outer one. Marsilius of Inghen's statement of a similar proof does not make clear that there are two circles involved from the start and that points A and B are probably on the two different circles: capiatur una linea trium punctorum que moveatur circulariter sic quod eius extremum circumscribat circulum: tunc quero quando punctus extremus A pertransit unum punctum, quantum pertransivit punctus B sibi immediatus. Si dicatur quod unum indivisibile: tunc sequitur quod circulus interior esset equalis superiori: tamen ambo circumscribuntur circa idem centrum: quod est impossibile. Si dicatur quod minus: igitur est dare minus indivisibili. Si majus: igitur circulus interior descriptus circa idem centrum esset major circulo exteriori quod est impossibile. Marsilius of Inghen, *op. cit.*, fol. 64^{v}, with corrections from Wadding's Scotus, III, p. 252. Cp. Murdoch and Synan, p. 254, paragraph 86.

[31] ad rem ymaginor istud quod, descripta aliqua superficie per circulos interiorem et exteriorem, tunc non potest protrahi linea recta ab alico puncto exterioris circuli ad centrum, quia semper duo puncta non directe cadunt contra se invicem, sicud alibi [the editors suggest "sed alibi" or "sicud alius alibi"] dico, igitur, quod casus est inpossibilis quia, licet una linea recta iuxta aliam semper habeat partem indivisibilem correspondentem parti alterius, tamen, si curventur et fiant circuli, punctus interioris non directe correspondebit puncto exterioris, sed duo interioris unico exterioris oblique. Istud patet quia, si recte linee fuerint equales in longitudine, et debeant fieri circuli, unus interior et alius exterior, oportet ad sensum auferre partem de altera ut fiat circulus interior immediatus exteriori; igitur, illa puncta, que directe correspondebant inter se quando fuerunt linee recte, modo non correspondent sibi directe quando fiunt circuli. Murdoch and Synan, pp. 259-60, paragraph 100. Nicholas of Autrecourt offers a similar defense. See his *Satis exigit ordo*, ed. J. R. O'Donnell, C.S.B., in: MS, 1 (1939), 213.

suggest that they are clearly predicated on unreliable data and hence false. Thus, according to Catton, this proof at least is invalid and there is still a possibility that lines may be composed of indivisibles.

But when he comes to the proof showing that the side of a square is commensurate with its diagonal, Catton relies upon an odd device used by several other Atomists, a plane geometry that becomes partly a solid geometry. This theory rests upon the idea that the geometrical shapes under consideration occupy space and thus have dimensions beyond what we might expect. For example, a surface has not only length and breadth but also a front and a back, and a line is thought to have both concavity and convexity. Thus a point or a line can lie in front of or behind another, even though they are apparently on the same plane surface. Catton suggests that the apparent commensurability of the side and diagonal of a square is not a valid proof because it is impossible to draw a straight line either from corner to corner of a square or from side to side. Instead there will be a wavy line, moving up and down apparently in a third dimension. It appears straight to our eyes, but is actually not straight at all.[32] Thus the apparent commensurability of the side and diagonal of a square are shown to rest on an illusion and to be false.

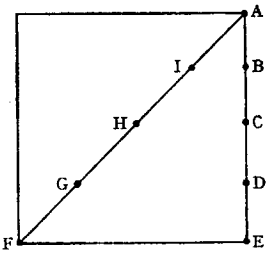
This mathematical oddity is matched by Nicholas Bonettus. After rehersing several of the mathematical proofs, which he properly attributes to "Algazeli", he comments that although it is difficult to disprove them, it is not impossible.[33] However, he offers only one set of

[32] Concedo quod est incommensurabilis coste in numero inpari. tunc, ad argumentum dico quod casus est inpossibilis, scilicet, quod protracta aliqua linea a costa in costam transversaliter, quod posset protrahi linea recta diametralis ab angulo in angulum, quia semper, propter obliquitatem situs, una pars, sive divisibilis, sive indivisibilis, in dyametro semper occurreret duabus partibus in linea recta. etiam [editors suggest "et"] si fiat dyameter ex partibus rectilinearum, dico quod non erit recta linea, ymo, a primo puncto angulari semper usque ad alium angulum erit linea tortuosa, semper punctum supra, de una linea recta, et statim post, punctum sub, de alia linea recta. et tanta longitudo resultabit in linea dyametrali, si ducatur ad rectitudinem supra lineam costalem, si ponatur iuxta eam quantam datam tortuositatem ille facte ex partibus rectilinearum, licet visus noster istas tortuositates non perpendat, quas tamen, in pisis, vel rebus divisibilibus contiguis, perpendere potest. Murdoch and Synan, pp. 261-62 paragraph 102. Gerard of Odo seems also to have shared the view that two dimensional figures sometimes entail three dimensions, although the context is different. See his views as reported by John the Canon in *Questiones Johannis Canonici*, Book VI, q. 1, recto side of a folio two folios after fol. t 4. See also Zoubov, *Walter Catton et al.*, for further illustration and discussion.

[33] Dissolutio autem istarum demonstrationum: licet sit difficilis: cum partim sint mathematica et partim naturalis: non tamen videtur impossibilis. Bonettus, *op. cit.*, fol. 80r.

mathematical proofs, five in all, seeking to show that the apparent commensurability of the side with the diagonal of a square is wrong as he has quoted it from al-Ghazzali. His refutation turns not on the principle but rather on apparent inconsistencies in the statement of the problem, which is to be sure his own, though he claims to be quoting al-Ghazzali.

First he raises the rather weak point that the problem as stated presupposes indivisibles of equal size. Bonettus rejects this, simply on the authority of "aliquos", in favor of the view that the indivisibles are of unequal size.[34] Then he notes a discrepancy in the number of points involved in the proof: al-Ghazzali had said a square of sixteen points, but actually his proof entails a square of twenty four points.[35] Next he criticizes al-Ghazzali's proof for the very thing it was designed to prove: namely, that the side and the diagonal are obviously not the same length and any assumption that implies that they are has to be wrong.[36] This refutation rests on the observation that a square with four points to a side will actually have five points on any given side, because the two corner points are shared with the neighboring sides:



Thus in the example shown, the four points on the diagonal, FGHI, correspond to only three points on the side, BCD. Bonettus claims

[34] quia supponit [sc. al-Ghazzali] quod aliqua magnitudo possit componi ex paribus indivisibilibus: quod non est verum secundum aliquos: sed tamen ex imparibus et ideo supponit oppositum dicti eorum: et ideo ratio videtur peccare. *ibid.*

[35] Supponit quaddratum componi ex 16 indivisibilibus et ipse utitur postea in quaddrato ex 24 indivisibilibus. Si non quelibet costa componatur ex quattuor indivisibilibus erunt 16 indivisibilia in quattuor costis et in quolibet diametro erunt quattuor: et sic 24 indivisibilia. *ibid.*

[36] Supponit quod quelibet costa sit composita ex quattuor indivisibilibus: et hoc est impossibile: quia licet diameter componatur ex quattuor indivisibilibus costa non componitur nisi ex tribus: quid patet: quia si ponatur quod quadratum cuius quodlibet latus est quattuor indivisibilium et describatur diameter: et terminetur ad costam: ex qualibet parte accipietur unum indivisibile: et sic non remanebunt nisi tria indivisibilia: in lateribus terminantibus alium diametrum si describerentur secundo in quadrato. *ibid.*

that point F would not correspond with point E, perhaps because he sees point E as belonging to the line EF.

The remaining two objections that Bonettus raises are of the same order: they challenge the proof but do not meet it on critical grounds. Whether this attests simply to a lack of mathematical acumen on Bonettus' part—not at all rare in the fourteenth century—or to his desire to build defenses for his theories I do not know. In any event I think that again one must grant that the Infinitists win the mathematical as well as the semantic victory.

When we come to the philosophical arguments, basically we find the Infinitists asserting the infinite divisibility of lines by one of three methods: Ockham's razor, Aristotle's arguments, and mathematical necessity. The Atomists struggle to get out from under the arguments in favor of infinity, but generally they fail to answer their opponents' challenges. Only Walter Catton has an original argument to offer. It is philosophically interesting, and I have not found any attempt to refute it.

Ockham's razor is really a special instance of the law of sufficient reason and was not invented by Ockham. Nonetheless, he and those who follow his teachings wield it with a ruthless skill. Basically, it boils down to the notion that entities should not be multiplied beyond necessity, or the simplest explanation that accounts for all the data is most likely to be right. In connection with the problem of the reality of points, both Ockham and Buridan shave things down so that there is no need to have points: everything can be satisfactorily accounted for without them.[37] Since this is the case, then it is more economical to explain reality without positing the existence of some unnecessary entity, like points.

The Infinitists also reproduced Aristotle's argument in several different varieties (see page 93 above). Adam Woodham, Gregory of Rimini, and John the Canon all reformulate the argument.[38] The notion

[37] Frustra fit per plura, quod potest fieri per pauciora; sed omnia, quae dicuntur de loco, possunt salvari absque hoc, quod locus sit res distincta a superficie corporis continentis, quia illa locabit, et ita continebit, et ita erit aequalis locato, sicut si ponatur alia res; igitur locus non est alia res a superficie. Ph. Boehner, ed., *The Tractatus de successivis Attributed to William Ockham*, Franciscan Institute Publications, 1, St. Bonaventure, N.Y. 1944, p. 70. Deinde ad principale arguo sic. Talia puncta non sunt ponenda res distincte a corporibus, si omnia aparentia sine eis possint salvari, quia in natura nihil est ponendum frustra. Sed sic est quod omnia possunt salvari sine his, ut apparebit solvendo rationem aliorum. John Buridan, *Quaestio de puncto*, II, 1, in Zoubov, *Jean Buridan*, p. 73.

[38] I cite the version of John the Canon. Although Woodham's is more economi-

of mathematical necessity, unlike the mathematical proofs cited earlier, is simply an indication of the mathematical need for fictions like the point. By affirming that points are a convenient, even a necessary, fiction for mathematicians, the Infinitists disarmed one sort of criticism that might be raised against them. Nicholas Oresme, John Buridan, and Gregory of Rimini all use this defense, particularly Buridan whose interest in science is well known.[39]

In trying to counter these arguments the Atomists attempted to establish essentially three paths to indivisibles: they tried to qualify infinity by suggesting that it was not really infinite; they attempted to work out a method of having extra points in a line; and Walter Catton tried to show that although dividing a thing infinitely may be possible, to do so destroys the nature of the thing being divided.

The qualification of infinity was first proposed by Robert Grosseteste and later picked up by Walter Catton and John Wyclif. It rests on the notion that infinity does not really exist but is a concession to

cally stated, it is also more accessible. Propositio ergo philosophi sic intelligitur: quod illa que se tangun<t> vel se tangunt [se] totis vel se tangunt secundum aliquid sui et aliquid non. Sed puncta non se tangunt secundum aliquid sui et secundum aliquid non: cum punctus sit indivisibilis et simplex, ergo tactus punctorum erit se totis et sic eorum tactus erit superpositio et sic per consequens non faciunt magnitudinem, quia omnis magnitudo habet partem extra partem. Ex superpositione enim non potest fieri magnitudo habens partem extra partem. Unde ex superpositione non potest fieri magnitudo in profundum. Cum ergo punctus sit indivisibilis secundum longum, latum, et profundum, tactus ipsius cum alio puncto non faciet magnitudinem, quia tactus eius cum alio puncto est superpositio. *Questiones Johannis Canonici*, Book VI, q. 1, recto side of a folio two folios after fol. t 4. Compare Murdoch and Synan, p. 268, paragraph 3, for Woodham, and *Gregorio de Arimino in secundo Sententiarum* (Venice 1503), dist. 2, q. 2, art. 1, fol. 32^{v}-33^{r}.

[39] Ideo oportet pro eius mensuratione ymaginare puncta, lineas, et superficies, aut istorum proprietates . . . . Etsi nihil sint puncta indivisibilia aut linee, tamen oportet ea mathematice sumere pro rerum mensuris et earum proportionibus cognoscendis. Nicholas Oresme, *De configurationibus qualitatum*, pars I, cap. 1, in MS. B.N. fonds lat. 14580, fol. 37^{v}, quoted by Zoubov, *Jean Buridan*, p. 54. Respondeo quod propter facilius loqui, mathematici imaginantur ac si essent puncta indivisibilia; et ita ponunt polos indivisibiles et centrum indivisibile. Sed secundum positionem naturalem et veram, polus est pars spherae et est corpus divisibile, scilicet circa quod aliae partes volvuntur vel circa quod aliae partes continentur; et sic de centro rotae. Et concedo quod illa pars quam tu capies tanquam polum iterum vertitur circulariter, et ideo potest sibi assignari polus, scilicet pars minor circa quam est illa pars, et sic in infinitum; nec est istud inconveniens, propter infinitam divisionem continui. Iohannis Buridani, *Quaestiones super libris quattuor de caelo et mundo*, ed. E. A. Moody, Mediaeval Academy of America publication, 40, Cambridge, Mass. 1942, Book I, *q.* 13, p. 59. Compare Buridan's statement in his comment on the *Physics*, *ed. cit.*, Book VI, q. 4, fol. 97^{r}-97^{v}, and Gregory of Rimini, *op. cit.*, dist. 2, q. 2, fol. 34^{r}.

human incapacity. We say that a line is infinitely divisible because we lack the time or the patience or the eyesight to divide it down to the ultimum punctum. But God can do it. And therefore, what is infinite to us is finite to him, and by means of his grace a human could be given the capacity to see the finite number of points on a given line.[40]

Another proposal favored by Walter Burleigh, Nicholas Bonettus, and accepted with qualifications by Catton is that between any two points there could always be found a third. This notion is part of what might be called the metaphysics of the line, its basic idea figuring in both Bonettus' and Catton's work. Bonettus proposes that the points of a line are of two sorts, those that are in act and those that are in potency. The only difference between the two is that those in act actually begin or end a given line while those in potency make up the rest of it.[41] However, the notion of a point's always existing between two other points is simply thrown in by the way, and Nicholas neither defends nor develops it.[42] Burleigh, however, offers the authority of Averroes for the theory.[43]

[40] See footnote 17 for quotations from Grosseteste, Catton, Wyclif, and John the Canon.

[41] preponamus distinctionem de puncto. Est namque in linea continua realiter punctus, scilicet punctus in actu et punctus in potentia . . . . At vero per punctum in actu intelligo punctum terminantem lineam et distinguentem et separantem. Quis actus est qui separat et distinguit. Nam sicut differentia formaliter vel fundamentaliter separat et distinguit unam speciem ab alia, sic punctus in actu unam lineam distinguit ab alia. Et talis punctus est ultimum linee . . . . Per punctum autem in potentia intelligo punctum coniugens partes linee adinvicem. Nicholas Bonettus, *op. cit.*, fol. 78^{v}-79^{r}. John the Canon is aware of the distinction, though he does little to explain it: quando linea dividitur: quero super quod cadit divisio . . . vel cadit inter duo puncta in potencia . . . *ed. cit.*, fol. t 4^{v}.

[42] Per punctum autem in potentia intelligo punctum coniugens partes linee adinvicem, quia inter duas partes semper est punctus in potentia medius. Nicholas Bonettus, *op. cit.*, fol. 79^{r}.

[43] Sed inter quecumque indivisibilia in eodem continuo est aliquod medium homogeneum, ergo in continuo non sunt indivisibilia consequenter entia ex quibus ipsum continuum componatur. Maior patet per diffinitionem consequenter entium. Et minor declaratur quia inter quecunque puncta est linea media et in qualibet linea est punctus, et ideo inter quelibet duo puncta est punctus, et per consequens homogeneum aliquod et sic de aliis indivisibilibus existentibus in aliis continuis ut de instanti tempore et de mutato esse in motu et sic de aliis. Quod autem inter quelibet duo puncta in eadem linea continua sit linea media non probat philosophus sed commentator probat illud dupliciter. Primo quod punctus non contiguatur cum puncto cum non habeat ultimum. Secundo quia puncta sunt ultima linearum: ergo impossibile est quod separatur a linea. Ergo inter quelibet duo puncta est linea media. *Burleus super octo libros physicorum*, Venice 1491, Book VI, q. 1, fol. v 3^{r}.

A view similar to this one and attributed to Henry of Harclay is attacked on logical grounds by Adam Woodham. According to Adam the proposition that there is a mediating point between the final point and the rest of the line is false because of a confusion between the universal and the particular. That is, the "point" and the "line" in the proposition are universals, whereas the "mediating point" is particular, and this confusion invalidates the argument.[44]

When Catton comes to defend the proposition that there is a point between the first point and the others on a line, evidently he had read Woodham's criticism or one like it. At any rate his defense takes into account the logical flaw that Woodham sees in the argument and applies to it the rule of universal instantiation: whatever is true of all is true of each.[45] Thus the "point" and the "line" and the "mediating point" are all made particular, and the proposition is then said to be true[46]. I have not found any further attempt to turn the balance in favor of the Infinitist position. At least in this area the contest seems to have been a tie.

[44] argumentum Harcle non cogit, cum in forma peccet penes figuram diccionis, et eciam penes fallaciam consequentis. Ideo fulciendo falsam opinionem que ponit huiusmodi indivisibilia, concedendum esset quod Deus videt omnia puncta in linea data, et quod videt primum punctum eius, et quod inter illum et quemlibet punctum alium huius linee intercipitur punctum aliquod et linea alia. Hec est universalis ex una parte et singularis ex alia, nec alica singularis ipsius falsa est. Unde ly 'punctum' et ly 'linea' sequencia mediate signum positum a parte subiecti supponunt confuse tantum. Sed hec que infertur et supponitur haberi ex iam concessa in posteriori deduccione, scilicet aliquod punctum est medium vel alia linea est media inter primum punctum linee et quodlibet eius punctum, hec, inquam, falsa est. Murdoch and Synan, p. 283, notes to paragraphs 37-39, quoting from another work of Woodham's on the same subject. On Henry of Harclay's views, see Fr. Pelster, *Heinrich von Harclay, Kanzler von Oxford, und seine Quästionen*, in: Miscellanea F. Ehrle, vol. I (= Studi e Testi, 37), Rome 1924, pp. 307-56, who has assembled much of the scattered evidence. Without citing a source he observes, "Dass nach Harclay das Continuum aus letzten unteilbaren Elementen besteht, wurde bereits erwähnt. Auch die aktuell unendliche Menge findet in ihm einen Verteidiger" (p. 339), evidently a notion Woodham would not have subscribed to.

[45] See D. P. Henry, *Medieval Logic and Metaphysics*, London 1972, p. 27.

[46] dicitur quod hec particularis vera est: *inter primum punctum et quodlibet aliud est aliquis punctus medius*, sed hec est falsa: *aliquis est punctus medius etc.* contra. licet predicatum stet confuse tantum, racione distribucionis, tamen, particulabo distribucionem, et stabit determinate, et tunc fiat conversio. Exemplum: *omnis homo est animal, igitur iste homo est hoc animal, vel illud.* similiter: *tota die est aliquis homo, igitur in hac parte diei est iste homo, vel ille.* sic in proposito, inter primam et hanc partem visam a deo est aliqua pars media, igitur aliqua pars est media inter primam et hanc partem, et sic de aliis, igitur sunt inmediate. Murdoch and Synan, p. 251, paragraphs 78 and 79.

We come finally to Catton's original argument against infinite divisibility, where he takes as his example of piece of meat. He argues, probably correctly, that there is a limit to the number of times the meat can be divided before its nature is altered or destroyed by the process. Therefore, he concludes, there is an indivisible particle of meat, and since this is so indivisibles exist.[47] Catton's proof may seem to an extent to beg the question, since he takes as his example a physical problem that suits his assumptions better than a line or a point. But in so doing he does nothing that his opponents do not also do, and the argument is stated in a convincing manner.

In a sense the difficulty illustrated by Catton's piece of meat is typical of the entire problem: each side seemed to miss the other's fundamental idea and argued against something else that it perceived. What the Infinitists were trying to show is that there are useful conventions in our language and other institutions, like mathematics and physics. But we should not automatically leap to the conclusion that a convention of length, like a yard, means that every physical thing having length is somehow made up of real yardsticks, or that the usefulness of a point means that any physical thing having length is somehow made up of a series of points.[48] But the Atomists seemed to see their opponents denying the nature of the physical reality their senses perceived in the world. Unprepared to see the problem in abstract terms, they were most reluctant to admit that there might in reality be an infinite number of anything. Hence Catton's argument

[47] capio istam carnem minimam quam dat naturalis et arguo sic ad intellectum eius: in carne minima non potest haberi operacionem naturalem competentem carni, igitur in illa minima carne non sunt infinite partes carnee, igitur est devenire ad indivisibile carnis. consequentia patet, quia contradiccio videtur quod in aliqua carne sint infinite carnes, sic quod per divisionem parcium eius ab eo nunquam potest sic consumi, quin infinita natura carnea manet, et tamen, in illa carne non possit natura exercere operationem convenientem carni. Murdoch and Synan, p. 240, paragraph 32.

[48] Buridan puts this very nicely: Alio modo punctum dicitur indivisibile quia dicitur punctum secundum quod est terminus alicuius linee ut si per ulnam volumus mensurare pannum, nos ponemus primam partem ulne supra primam partem panni ita quod neutra a parte ante excedit reliquam et videmus quantum se extendit ulna super pannum secundum panni longitudinem. Et hoc erit ulna panni. Consequenter immediater post illam ulnam panni nos iterum ponemus ulnam que est mensura et erunt due ulne panni et sic deinceps et sciemus quod pannus est longitudinis viginti quattuor ulnarum. Consimiliter autem imaginando nos ponemus punctum primum ulne secundum imaginationem super primum punctum panni, et videmus quantum se extendit ulna super pannum et sic consequenter revertitur idem quod prius ideo licet uti illa imaginatione. Commentary on the *Physics*, *ed. cit.*, Book VI, q. 4, fol. 97^{v}.

about the piece of meat: for him it is not a matter of convention but of actuality.[49]

Murdoch and Synan claim that the argument over the nature of the continuum, whose beginnings they date around 1313, was substantially over by 1335, and this is probably true if our criteria include the stipulation that new arguments and new insights should develop as the discussion progresses. The fact is that we do find philosophers in the second half of the century—John Wyclif, Marsilius of Inghen, and Albert of Saxony to mention only three—who continue the discussion as if it were still an issue. But it is not. Marsilius and Albert, both pupils of Buridan's and like him Infinitists, marshall all the proper arguments, with Albert being more metaphysical and Marsilius more mathematical. But the fact that they are the proper arguments, the arguments that had been well used fifty years earlier, suggests strongly that we are no longer dealing with a vibrant controversy.

Wyclif, whose treatment of the problem comes in the ninth chapter of his *Logic*, is particularly annoying. He takes an Atomist's point of view and states his propositions as if all the opposition to them had been frivolous or non-existant. He asks what the "moderni" (always a term of opprobrium in the fourteenth century) would call a line, a surface, or a point, as if Ockham, Buridan, Oresme and others had never proposed and defended the notion of a useful fiction.[50] He restates Grosseteste's idea that God can know how many points there are on a line, but he presents no proofs nor does he even acknowledge that there are other views on the matter.[51] He revives the Augustinian distrust of knowledge gained through the senses,[52] despite the discus-

[49] Compare this argument of Catton's, which seems to anticipate Heisenberg's uncertainty principle: Ad primum principale argumentum dico quod [indivisibilia componentia continuum sunt] finita, tamen per principale: causam naturalem non potest compleri tota divisibilitas continui huius, set antequam deveniret ad minima corrumperet ea. Murdoch and Synan, p. 246, note to paragraphs 56 and 57, square brackets theirs.

[50] Libenter tamen scirem a modernis argumentibus geometrice contra dictam viam de compositione continui ex non quantis, quid ipsi vocant lineam, superficiem, punctum et angulum; si negent talia mathematicalia posse esse, quomodo non verentur arguere, peccando in materia, dum sumunt sibi notorie impossibilia. *Op. cit.*, pp. 45-46.

[51] Novit [sc. Deus] eciam in qua proporcione quicunque numerus punctorum se habet ad alium; et de istis habemus nos inseparabilem ignoranciam. *Ibid.*, p. 36.

[52] Pro quo declarando, notandum secundum sententiam perspectivorum et communem experientiam, quod maxima decepcio contingit in sensibus, in significando continuitatem motus. Cuius ratio est, quia tam ad situs quam ad motus significacionem requiritur tempus conveniens; et ideo parvitas temporis facit

sions of the past seventy years on intuitive and abstractive cognition. Although Wyclif's idiosyncracies give him his own peculiar flavor, still it seems as evident for him as for Marsilius and Albert that the time of controversy is over.

Yet it is masters like Wyclif or Albert or Marsilius with whom the *Patience*-poet would have had to study. If the dating that places him in the second half of the fourteenth century is correct, and I know of no substantial objection to it, then there would have been no one else for him to work with. He too then enters the lists at a time when the battle has moved elsewhere, and the lectures he would have heard would no doubt have had the same second-hand sound to them as the discussions of Marsilius, Albert, and Wyclif. Yet such a time has its own advantages, for when tradition is worn out then its seams show most clearly. Such seams reveal the structural strengths and weaknesses of the construction, so that an observant man can both use and criticize the material he finds. Jean de Meun uses Guillaume de Lorris' courtly love in this manner, and later Chaucer uses Jean de Meun's encyclopedic poem in a similar way. The *Patience*-poet in his turn seizes certain parts of the material at hand in the lecture room and, playing the part of an Atomist, transforms them to his own ends.

But what in particular does he seize upon? For the most part what he takes from the lecture hall are fairly simple notions: that a point is indivisible and real, that it forms the end and the beginning of a line, that it is in itself without dimensions. All the rest is his own embroidery on this simple cloth. The poem, then, reflects little of the controversial aspects of the problem. But it does develop the terms of the discourse in a new direction, both moral and artistic, which provides considerable delight and might well have caused one of the late fourteenth-century masters some exasperation to refute.

The poet insists that patience is a point, not merely by means of the repetition in the first and last lines of the poem, but also by stressing its place as the last of the eight beatitudes and thus the point with which the line of beatitudes ends.[53] And since the poet insists upon the similarity of the first and last beatitudes,

quolibet illusiones: ut ticio circumductus in noctibus videtur causare ignitum circulum, sicut et punctus sensibilis rubricatus in base alta troci celeriter circumducti. *Ibid.*, p. 40.

[53] It is worth observing that discussions of the continuum, both early and late in the century, were sometimes attached to discussions of theological issues. Ockham goes into the problem at some length as a prelude to discussions of the eucharist in the *De sacramento altaris*, and Gregory of Rimini introduces it in his

For in þe tyxte þere þyse two arn in teme layde,
Hit arn fettled in on forme, þe forme and þe laste,
And by quest of her quoyntyse enquylen on mede,
And als, in myn vpynyoun, hit arn of on kynde. (37-40)

the two can be thought of as points, the one that begins the series and the one that ends it. There is also, as Jay Schleusener has noted, Augustine's view that the beatitudes are circular in form, a seven-rung ladder to perfection where the eighth rung returns to the beginning, thereby crowning a process that leads toward perfection and at the same time recommencing the journey on a higher level.[54] This idea of a straight line that can also be a circle is mentioned by Thomas Bradwardine in his discussion of the continuum, where he is more precise physically than Augustine, making one point of the line fixed, the other moving about it in the manner of a compass.[55]

Through his use of spatial imagery the poet also develops the image of the point in a less philosophical direction. As the story unfolds in the poem, Jonah is seen to be acutely aware of his own tiny helplessness before God's cosmic indifference. His only defense, as he sees it, is to take shelter from God, but the succession of shelters, the ship, the whale, and the woodbine hut, are all unsatisfactory and must give way to God's might. Through these images the poet demonstrates that defense against God is needless and that trust in his omniscient goodness will allow one to become a tiny speck in cosmic space, a "mote in at a munster dor" (268), and not fear the consequences.[56] Thus patience

commentary on the Sentences as a preface to the question "in quo loco fuerunt angeli creati". Richard of Middleton includes it as part of his commentary on the divine and human natures of Christ. Wyclif too, in the midst of the logic of the continuum, introduces by way of analogy the moral issue of perseverance in an act of merit. It is not farfetched for the poet to connect a point with a virtue.

[54] "*Patience*, lines 35-40", *MP*, 67 (1969), 64-66. Schleusener refers to Augustine's *De sermone domini in monte*, *PL*, 34, 1234-35. The same idea is mentioned in Peter Cantor, *Verbum Abbreviatum*, *PL*, 205, col. 298 B: Septem tantum sunt beatitudines perfectae consummantes scalam Jacob (Gen. xxviii), de quibus iam diximus; verum praeter has restat patientia, quae octava redit ad caput circumeundo, purificando, probando et manifestando omnes virtutes.

[55] Cuiuslibet recte linee finite uno termino quiescente potest reliquus eius terminus circulariter uniformiter et continue circumferri, tota recta et qualibet parte eius magna ad terminum eius immobilem terminata circulum describente et quolibet eius puncto moto circumferentiam circuli faciente. Thomas Bradwardine, *De continuo*, quoted in Marshall Clagett, *The Science of Mechanics*, p. 233.

[56] I have argued this interpretation at greater length in "Sheltering Space and Cosmic Space in the Middle English *Patience*", forthcoming in *Annuale Medievale* for 1980.

is a point in another sense: it is an emblem of the dimensionless virtue of patience, the virtue of resignation and trust, which the poem exemplifies.[57]

And this, I believe, is where the poet's strongest emphasis lies, on patience as one of the passive virtues of the beatitudes. He insists, first of all, that both patience and poverty are inseparable from the line of which they are the points:[58]

> For þer as pouert hir proferes ho nyl be put vtter,
> Bot lenge where-so-euer hir lyst, lyke oþer greme;
> And þere as pouert enpresses, þaȝ mon pyne þynk,
> Much, maugre his mun, he mot nede suffer.
>
> Thus pouerte and pacyence arn nedes play-feres;
> Syþen I am sette with hem samen, suffer me by-houes; (41-46)

The virtues exemplified by the beatitudes are all virtues of resignation and acceptance, rather than more active virtues, such as the charity of the good Samaritan. These are the virtues that exemplify modes of waiting and suffering: to be pure in heart, to be meek, to weep, to hunger after righteousness, to be merciful, to be clean of heart, to be peaceful, to be patient. None of these envisions the sort of aggressive virtue that moved Jesus to drive the money changers from the temple.

The poet is fully aware of this passive quality, as he makes clear in

[57] The dimensionlessness of a point does not occupy the philosophers to any great extent, not even the Infinitists for whom it was true that a point had no dimensions. Nonetheless, they are aware of it; see, e.g., Gregory of Rimini: sed ut ubi fingit esse punctum, intelligatur negari omnis magnitudo. *Op. cit.*, d. 2, q. 2, fol. 34^{r}.

[58] This aspect of inseparability was also discussed but was not so common as other aspects of the problem. For example, Walter Burleigh maintains that the terminating point of a line is something different from the rest of the line: Nullum non habens ultimum dividitur in ultimum et habens ultimum. Hoc statim patet quod ultimum aliud est ab eo cuius est ultimum. Sed omne indivisibile est non habens ultimum: ergo nullum indivisibile dividitur in ultimum et in habens ultimum. Minor huius rationis patet quia ultimum aliud est ab eo cuius est ultimum. Et ita omne habens ultimum est divisibile; et per consequens omne indivisibile est non habens ultimum. *Burleus super octo libros Physicorum*, fol. v 3^{r}. Wyclif also maintains their inseparability: aliter contingeret auferri punctum a linea continuata cum alia, non plus auferendo; et per idem quemlibet punctum linee contingit auferri, et sic omnes, cum hoc quod maneat eadem linea. *Op. cit.*, p. 30. Ockham does the same: punctus est nomen vel intentio importans non ulteriorem pertransitionem secundum longitudinem . . . et ideo linea numquam finitur alia re sed se ipsa finitur formaliter. *Expositio Aurea*, quoted in Moody, *The Logic of William of Ockham*, p. 147.

playing with two of the English meanings of *patior*: to have patience and to suffer. As he says in the prologue:

> Thus pouerte and pacyence arn nedes play-feres;
> Syþen I am sette with hem samen, suffer me by-houes; (45-46)

Yet at the same time he also makes clear that patience is not synonymous with a collapse into indiscriminating sloth. It may require both standing and waiting and a more active response:

> ȝif me be dyȝt a destyne due to haue,
> What dowes me þe dedayn oþer dispit make?
> Oþer ȝif my lege lorde lyst on lyue me to bidde
> Oþer to ryde oþer to renne to Rome in his ernde. (49-52)

What active and passive responses have in common is the acceptance without complaint or avoidance of whatever duty one's lord imposes. And it is this passive willingness (along with its opposite), and not mere passivity, that the poet illustrates in the story of Jonah.

The quality of passive acceptance is less a matter of quietism, more a means of distinguishing between the human and divine exercise of virtue. It is characteristic of divine virtue to initiate, of human to do the will of the divine. Thus God decides that the behavior of the Ninevites is insufferable and that Jonah must be sent to warn them of God's impending judgment:

> 'Rys radly', he says, 'and rayke forth euen;
> Nym þe way to Nynyue wyth-outen oþer speche,
> And in þat cete my saȝes soghe alle aboute,
> Þat in þat place, at þe poynt, I put in þi hert.
>
> For iwysse hit arn so wykke þat in þat won dowelle,
> And her malys is so much, I may not abide, (65-70)

And it is God too who sends the storm to stop Jonah's flight from his duty:

> For þe welder of wyt þat wot alle þynges,
> Þat ay wakes and waytes, at wylle hatȝ he slyȝtes.
> He calde on þat ilk crafte he carf with his hondes;
> Þay wakened wel þe wroþeloker, for wroþely he cleped. (129-32).

Again, God is responsible for Jonah's safety in the whale's belly (261), and the whale only vomits Jonah onto the beach when God commands

him to do so (337-38). It is God's decision and his only to spare the Ninevites (407-8), and he is the sole cause for both the growth and the destruction of the woodbine hut that temporarily shelters Jonah on the plains outside Nineveh (443-44 and 467-72).

For earthly creatures patience is the corresponding virtue by means of which one accepts the will of God. The best illustration of it is perhaps the response of Eurus and Aquilon, the two winds God commands to cause the storm at sea:

> Þenne watȝ no tom þer bytwene his tale and her dede,
> So bayn wer þay boþe two his bone for-to wyrk. (135-36)

Almost as prompt to God's will is the response of the Ninevites to Jonah's preaching:

> Þis speche sprang in þat space and spradde alle aboute
> To borges and to bacheleres þat in þat burȝ lenged;
> Such a hidor hem hent and a hatel drede,
> Þat al chaunged her chere and chylled at þe hert. (365-68)

Jonah contrasts sharply with the Ninevites and the winds. He requires an unusually severe lesson at sea before he will do God's bidding and a tutorial with God himself before he is even disposed to accept God's mercy and restraint with respect to the Ninevites. In fact he exemplifies patience directly only once in the poem, just after the whale has deposited him on the beach:

> Þenne a wynde of Goddeȝ worde efte þe wyȝe bruxleȝ:
> 'Nylt þou neuer to Nuniue bi no kynneȝ wayeȝ?'
> 'ȝisse, lorde', quoþ þe lede, 'lene me þy grace
> For-to go at þi gre; me gayneȝ non oþer'. (345-48)

Traditionally, patience is opposed to wrath,[49] and the *Patience*-poet does not gainsay this opposition (cp. lines 410-11, 481, 491-92, 497, etc.). But his emphasis lies on other characteristics, namely the contrast between the divine virtues that initiate and the human that accept. I suggest that the point is a superb image for this human virtue of willing acceptance, primarily because like the Atomists' point, patience is an indivisible virtue. In effect it is that fundamental virtue

[49] See the sections on anger and the remedy for anger in Chaucer's "Parson's Tale" in F. N. Robinson, ed., *The Works of Geoffrey Chaucer*, 2nd ed., Boston 1957, pp. 244-49. The contrast apparently originates with Prudentius, *Psychomachia*, lines 109-77.

that underlies all others, the acceptance of God's will.[60] It may be that God will also require the translation of acceptance into other more active virtues, such as preaching to the Ninevites or helping a man wounded by thieves. But it is, as the poet sees it, the irreducible root virtue on which all the others depend. In the sense of indivisibility then, it is a point.

It is also a point in that it is a virtue that willingly exposes human vulnerability and helplessness. Man's tiny stature, compared with God's vast and remote grandeur, leads Jonah to feel that he is no more than an indifferent speck on the remote horizon. Yet although the poet never alters the relative proportions of man and his universe, he does demonstrate that God never regards such a tiny speck with indifference. Thus the virtue of patience is exemplified in the patient man, who is as tiny a point as a mote in a minster door but is nonetheless always in God's watchful view.

One can imagine the serious scholastic Infinitist, Buridan say, replying to all this: "We are not talking about virtues". But in effect what the poem has done is to leave him with a paradox. It has first made a point into something real, and the good scholastic cannot respond, even if he is a little cross, that virtues are not real. Of course, the poem slips well outside the bounds of the philosophical discourse, and the good scholastic could show how it has done so, if he had the time. He could also solve the apparent paradox without denying the reality of virtue. But for the moment of the poem, the debaters' terms, grown sterile with repetition, are charged with a new content and a new life.

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[60] Patience as the fundamental human virtue may also have originated with Prudentius. At the end of the battle between Ira and Patientia, he says:

> Nulla anceps luctamen init uirtute sine ista
> uirtus; nam uidua est, quam non Patientia firmat.

Aurelii Prudentii Clementis, *Psychomachia*, lines 176-77, in: *Carmina*, ed. M. P. Cunningham, Corpus Christianorum 126, Turnholti 1966, p. 157. Deguileville too elaborates this fundamental sense of patience; see John Lydgate, tr., *The Pilgrimage of the Life of Man*, ed. F. J. Furnivall, EETS. ES. 77, 83, and 92, London 1899, 1901, and 1904, lines 7293-7516, where the pilgrim's body armor, designed to bear all blows, is identified as Patience. Cp. also W. Nelson Francis, ed., *The Book of Vices and Virtues*, EETS. OS. 217 (London: Oxford 1942), p. 167. See also the detailed discussion in F. N. M. Diekstra, *Jonah and* Patience: *the Psychology of a Prophet*, in: ES, 55 (1974), 205-17.